

## MODELLING ANIMAL SYSTEMS RESEARCH PAPER

# The development of a dynamic, mechanistic, thermal balance model for *Bos indicus* and *Bos taurus*

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### SUMMARY

The dynamic model presented in the current paper estimates heat production and heat flow between growing and mature cattle (*Bos indicus* and *Bos taurus*) and the surrounding environment. Heat production was calculated using the NRC (2000) and heat flows between the animal and the environment were based largely on existing models and physical principles. Heat flows among the body core, the skin, the coat and the environment were calculated. Heat flows from and to the environment included solar radiation, long wave radiation, convection and evaporative heat loss. Physiological responses of cattle (sweating, panting and vasodilation) were modelled through mechanistic equations. The model required weather (radiation, temperature, wind and vapour pressure), animal (body-core weight and genotype-specific parameters) and dietary inputs (dry matter intake rates and diet composition) and estimated heat balance and the physiological responses of the animal to within-day weather variation. The current paper has focused on heat stress, although the model was designed to run under both hot and cold climatic conditions. The model developed in the current paper provides researchers and livestock producers with the ability to predict heat stress and to evaluate mitigating procedures.

### INTRODUCTION

In cattle, thermal stress from either heat or cold may result in large losses in production. Physiological responses to heat stress include cutaneous vasodilation, sweating, increased respiration and decreased feed intake. An initial reaction to increased environmental temperature is vasodilation, which increases blood flow to the surface of the skin to bring warm blood in close proximity to the skin surface and thus increase heat loss through convection (Finch 1985). Increases in sweating and respiration rate also increase latent heat loss in the animal (Allen 1962; Thomas & Pearson 1986; Gaughan *et al.* 1999). Ultimately, if physiological processes cannot compensate for the excess body-core heat, a decrease in heat production can be achieved through decreased feed intake (Brown-Brandl *et al.* 2003; Beatty *et al.* 2006). Increasing heat loss through increasing vasodilation and respiration

and sweating rates also increases the maintenance requirement for the animal, and decreasing feed intake further limits the amount of energy and nutrients directed towards tissue growth or milk production. The increased maintenance requirement and decreased intake lead to a decrease in animal performance (Blackshaw & Blackshaw 1994; Mitloehner & Laube 2003), which results in an economic loss for producers.

Because of the economic losses associated with heat stress, temperature–humidity indices (THI) were created to estimate level of heat stress of the animal. The THI, proposed by Thom & Bosen (1959), takes into account temperature and humidity to determine the level of heat stress for the animal. However, these indices do not account for physical processes behind the phenomena and therefore have limited capacity in predicting dynamic changes in body-core temperature, physiological and production responses and in evaluating different mitigation options. Therefore, a model that simulates heat flow between the animal and its environment, and heat production by the

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animal, can be useful to understand mechanisms of thermal stress, improve prediction of losses in animal performance and, therefore, evaluate the effectiveness of possible management interventions.

The objective of the current work was the creation of a dynamic, mechanistic model to:

- estimate major flows of heat into and out of a bovine
- use heat balance to calculate changes in body-core temperature for growing and mature *B. indicus* and *B. taurus* in response to climatic factors: air temperature, relative and vapour pressure, radiation/shade and wind.

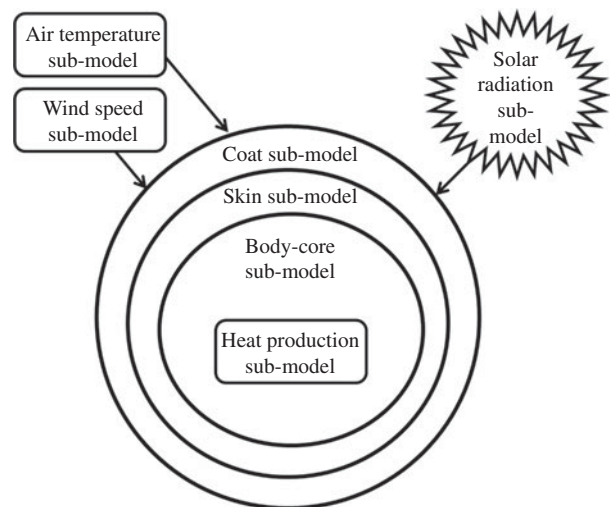
## MATERIALS AND METHODS

### Model design

A dynamic model, rather than a steady-state model such as that of Turnpenny *et al.* (2000a, b), was developed since experimental results (e.g. Brown-Brandl *et al.* 2005) have shown that variables such as body-core temperature and respiration rate lag behind changes in environmental variables such as ambient air temperature and incident solar radiation by several hours.

The heat balance model is a three-node model with three state variables: heat contents for body core, skin and coat. Figure 1 shows an overview of the model with all of the flows. A number of three-node heat transfer models have appeared in the literature over the years (see, for example, Porter & Gates 1969; Vera *et al.* 1975; Bakken 1976; McGovern & Bruce 2000). Similar multi-node models have appeared in the human thermal model literature, for example: Stolwijk & Hardy (1966), Gagge *et al.* (1971) and Huizenga *et al.* (2001). The present model is similar to that developed by Gagge *et al.* (1971), which is also known as the Pierce model (Gagge & Gonzalez 2010). The Pierce model is often called a two-node model, although in reality it contains a steady-state energy balance for clothing (for cattle: coat) temperature (Doherty & Arens 1988). The Pierce model continues to be applied in problems such as automobile comfort (Kaynakli & Kilic 2005) and exercise physiology (Takada *et al.* 2009).

All variables, parameters and inputs in the heat balance model are defined in Table 1. The model has three equations (Eqns 1.0, 2.0 and 3.0, in Tables 2 to 4, respectively), with the first two being differential equations and the third being an algebraic equation, and three unknowns: temperature of the body core,



**Fig. 1.** Overview of the Thompson model. The three concentric circles represent the three layers of the animal: the body-core, skin and coat sub-models. The heat production sub-model is within the body-core. The solar radiation sub-model gives heat to the coat sub-model. The air and wind speed sub-models directly affect all three layers of the animal.

skin and coat ( $T_b$ ,  $T_s$  and  $T_c$ , respectively, K). Equation 1.0 represents the change in heat content in the body core (Table 2), Eqn 2.0 represents the change in heat content in the skin layer (Table 3) and Eqn 3.0 represents the heat content balance in the coat (Table 4). Equation 3.0 is similar to that proposed for clothing by Voelker *et al.* (2009) without the evaporation terms. The heat balance model assumes that the storage term in the coat energy balance is small and can be neglected. This is similar to the assumption made for clothing in the early human thermal models (Huizenga *et al.* 2001). Thus, the change in heat content for the coat layer (Eqn 3.0) is set to zero. All of the heat fluxes on the right-hand side of Eqns 1.0, 2.0 and 3.0 are either functions of  $T_b$ ,  $T_s$  and  $T_c$  or known functions of time under ambient weather conditions. Heat transfer from appendages (head and legs) is accounted for through skin and coat surface area. The heat transfer through the skin and coat are multiplied by the total surface area, which includes appendages and the dewlap. The assumption is that the resistance within the coat, skin and body core are uniform and thus an average value can be multiplied by the total surface area to represent total heat flows.

Model energy balances were derived using a 'physical formulation' of the finite-difference method for one-dimensional transient conduction problems (Myers 1971). The model assumes that the temperatures in the body core, skin and coat layer are

Table 1. Description of all variables, parameters and inputs used in the model

Symbol	Name	Units
$A$	Surface area of animal	$m^2$
$a$	Time lag in maximum wind speed after noon, 1·24 (Hasson <i>et al.</i> 1990)	h
$A_h$	Shadow area cast by animal	$m^2$
$a_{rr}$	Slope for respiration rate equation	breaths/K/min
$a_{sr}$	Parameter for sweating rate equation	$g/m^2$
$B$	Variable accounting for Julian day in EoT	rad
$b$	Coefficient that controls wind speed decrease at night, 3·59 (Hasson <i>et al.</i> 1990)	Dimensionless
$b_A$	Animal surface area parameter	$m^2/kg^{0.57}$
BBD	Time from sunrise	h
$b_{rr}$	Intercept for respiration rate equation	breaths/min
$b_{sr}$	Parameter for sweating rate equation	$K^{-1}$
$c$	Time lag for minimum wind speed after sunrise, 1 (Hasson <i>et al.</i> 1990)	h
$clf$	Cloud cover (index ranging from 0–1)	Dimensionless
cos	Cosine (function)	–
$cp$	Wind penetrability of the coat, $9.1 \times 10^{-6}$ (Campbell <i>et al.</i> 1980)	m
$C_{pb}$	Specific heat capacity of the body core, 3·4 (Stolwijk & Hardy 1966)	J/kg/K
$C_{pc}$	Specific heat capacity of the coat	J/kg/K
$C_{pr}$	Penetrability of the coat by the wind	m
$C_{ps}$	Specific heat capacity of the skin, 3·47 (Stolwijk & Hardy 1966)	J/kg/K
$c_t$	Empirical parameter to calculate temperature, 0·39 (Cesaraccio <i>et al.</i> 2001)	Dimensionless
$d$	Diameter of body-core	m
$D$	Diffusion coefficient of water vapour at 293·15 K, $2.5 \times 10^{-5}$	$m^2/s$
DDY	Time length in which wind speed is calculated during the day	h
$D_j$	Julian day	d
$D_l$	Day length	h
DMI	Dry matter intake, 0·02 kg Intake/kg $M_b$ .	kg/d
$DMI_g$	Dry matter intake for gain	kg/d
$DMI_m$	Dry matter intake for maintenance	kg/d
DWT	Daily wind travel	m/s
e	Natural exponential (function)	–
EBW	Empty body weight	kg
$elev$	Elevation, 500	m
EoT	Equation of time	min
$f_c$	Form factor	Dimensionless
$Gr$	Grashof number	Dimensionless
$H$	Height of the atmosphere, 8000	m
$H_0$	Sunset hour for current day	h
$H'_0$	Sunset hour for previous day	h
$H_d$	Half-day length	rad
$H_dE$	Heat of digestion	W
HE	Heat production	W
$H_eE$	Endogenous heat production (fasting heat production)	W
$h_m$	Height at standard wind speed measurement, 10	m
$H_n$	Sunrise hour for current day	h
$H_p$	Sunrise hour for following day	h
$hr$	Hour of the day	h
$H_rE$	Heat energy from retained energy (growth)	W
$h_x$	Mid animal height	m
$H_x$	Time of maximum temperature for current day	h
$H'_x$	Time of maximum temperature for previous day	h
$J_o$	Solar constant, 1350 (Johnson 1954)	$W/m^2$
$k_a$	Thermal conductivity of air, 0·0262	W/m/K
$k_{maint}$	Maintenance coefficient ( $8.91 \times 10^{-7}$ ; NRC 2000)	$W/kg EBW^{0.75}$

Table 1. (Cont.)

Symbol	Name	Units
$l$	Coat length	m
$\ln$	Natural log (function)	–
$longd$	Longitude, 120.5 (Duffie & Beckman 1991)	degrees
$l_w$	Wind penetration of the coat	m
$m$	Optical air mass number	Dimensionless
$M_b$	Body-core mass	kg
$M_c$	Coat mass	kg
ME	Metabolizable energy	J/kg DM
$M_s$	Skin mass	kg
$NE_g$	Net energy for gain	J/kg DM
$NE_m$	Net energy for maintenance	J/kg DM
$N_l$	Night length	h
$Nu$	Nusselt number	Dimensionless
$P$	Atmospheric air pressure	Pa
$p$	Coat penetration parameter, 1800 (Turnpenny <i>et al.</i> 2000a)	$m^{-1}$
$P_o$	Atmospheric pressure at sea level, $1.01 \times 10^5$	Pa
$p_w(T_a)$	Vapour pressure at air temperature	Pa
$p_{w,sat}(T)$	Saturation vapour pressure at temperature $T$ (function, Eqn 1.7)	Pa
$p_{w,sat}(T_a)$	Saturation vapour pressure at air temperature	Pa
$p_{w,sat}(T_b)$	Saturation vapour pressure at body temperature	Pa
$p_{w,sat}(T_s)$	Saturation vapour pressure at skin temperature	Pa
$Q_{10}$	Ratio of heat production when temperature elevated by 10 K, 2.0 for ungulates (McArthur 1987)	Dimensionless
$q''_{cond, b-s}$	Heat flow through conduction from body-core to skin	$W/m^2$
$q''_{cond, s-c}$	Heat flow through conduction from skin to coat	$W/m^2$
$q''_{conv, c-a}$	Heat flow through convection at coat surface	$W/m^2$
$q''_{evap, s-a}$	Heat flow through cutaneous evaporation	$W/m^2$
$q''_{lw rad, c-a}$	Heat flow through long wave radiation	$W/m^2$
$q''_{resp, b-a}$	Heat flow through respiratory loss	$W/m^2$
$q''_{solar, a-c}$	Heat flow through solar energy absorption	$W/m^2$
$r'_R$	Resistance of radiation	s/m
$r_b$	Resistance of conduction; 400 (Turnpenny <i>et al.</i> 2000a)	s/m
$r_{ch}$	Resistance through coat	s/m
$r_{CH0}$	Resistance through coat with no wind	s/m
$r_d$	Resistance of free convection	s/m
$R$	Correction factor for variation of earth-sun distance throughout the year	rad
$R_{diff}$	Diffuse radiation	$W/m^2$
$R_{dir}$	Direct radiation	$W/m^2$
$Re$	Reynolds number	Dimensionless
$R_g$	Radiation reaching the ground	$W/m^2$
$R_{gd}$	Daily global radiation	$J/m^2/d$
$R_{gh}$	Hourly radiation reaching the earth's surface	$W/m^2$
$r_H$	Resistance to convection	s/m
$R_{od}$	Daily extraterrestrial radiation	$J/m^2/d$
$R_{oh}$	Hourly extraterrestrial radiation	$W/m^2$
RR	Respiration rate	breaths/s
$RR_{min}$	Respiration rate minimum (12, compiled data)	breaths/min
$r_s$	Resistance to heat transfer from body-core to skin	s/m
$r_{s,max}$	Maximum resistance to heat transfer from body-core to skin ( $36.6 \times M_b^{0.33}$ ; Bruce & Clark 1979)	s/m
$r_{s,min}$	Minimum resistance to heat transfer from body-core to skin (19, Finch 1985)	s/m
$r_v$	Resistance to cutaneous water vapour transfer	s/m
$r_{vl}$	Resistance to respiratory water vapour transfer	s/m
sin	Sine (function)	–

Table 1. (Cont.)

Symbol	Name	Units
$SR$	Sweating rate	$\text{g/m}^2/\text{h}$
$SSN$	Wind speed at sunset	$\text{m/s}$
$St$	Standard time	$\text{h}$
$stdmer$	Standard meridian for local time zone ( $120^\circ$ for Davis, CA)	degrees
$T$	Temperature	$\text{K}$
$t$	Time	$\text{s}$
$T(t)$	Air temperature at a given time ( $t$ )	$\text{K}$
$T_0$	Temperature at sunset for current day	$\text{K}$
$T'_0$	Temperature at sunset for previous day	$\text{K}$
$\tan$	Tangent (function)	–
$T_a$	Air temperature	$\text{K}$
$T_a(St)$	Air temperature at time $St$ (function)	$\text{K}$
$T_b$	Body core temperature	$\text{K}$
$T_{bref}$	Reference body-core temperature, 311.65	$\text{K}$
$T_c$	Coat temperature	$\text{K}$
$T_{dp}$	Dew point temperature	$\text{K}$
$T_g$	Radiant temperature of the ground	$\text{K}$
$timediff$	Difference between solar and standard time	$\text{h}$
$T_{mid}$	Average of coat and skin temperatures	$\text{K}$
$T_n$	Minimum temperature for current day	$\text{K}$
$T_p$	Minimum temperature for following day	$\text{K}$
$T_r$	Radiant temperature of surroundings	$\text{K}$
$T_s$	Skin temperature	$\text{K}$
$T_{sky}$	Radiant temperature of the sky	$\text{K}$
$T_{va}$	Virtual temperature of the air	$\text{K}$
$T_{vb}$	Virtual temperature of the body-core	$\text{K}$
$T_{vdif}$	Difference in virtual temperatures between skin and air	$\text{K}$
$T_x$	Maximum temperature for current day	$\text{K}$
$T'_x$	Maximum temperature for previous day	$\text{K}$
$u$	Wind speed at mid-animal height	$\text{m/s}$
$u_m$	Wind speed	$\text{m/s}$
$V_t$	Tidal volume	$\text{l/breath}$
$WSN$	Minimum wind speed	$\text{m/s}$
$WSX$	Maximum wind speed	$\text{m/s}$
$\alpha$	Altitude angle	$\text{rad}$
$\beta$	Parameter accounting for body curvature	Dimensionless
$\gamma$	Psychrometer constant, 66	$\text{Pa/K}$
$\delta$	Solar declination	$\text{rad}$
$\epsilon_c$	Coat emissivity, 0.98 (da Silva 2000)	Dimensionless
$\epsilon_g$	Ground emissivity, 0.98 (da Silva 2000)	Dimensionless
$\epsilon_r$	Environment emissivity	Dimensionless
$\epsilon_{sky}$	Sky emissivity	Dimensionless
$\theta$	Solar zenith	$\text{rad}$
$\lambda$	Latent heat of vaporization, 2430	$\text{J/g}$
$\mu$	Dynamic viscosity of air	$\text{N s/m}^2$
$\pi$	Pi (3.14159)	–
$\rho$	Density of air	$\text{kg/m}^3$
$\rho_c$	Coat reflection coefficient (solar radiation)	$\text{J reflected/J intercepted}$
$\rho C_p$	Specific heat of air, 1220	$\text{J/m}^3/\text{K}$
$\rho_g$	Ground reflection coefficient, 0.25 for grass (da Silva 2008)	$\text{J reflected/J intercepted}$
$\sigma$	Stefan–Boltzmann constant, $5.669 \times 10^{-8}$	$\text{W/m}^2/\text{K}^4$
$\tau$	Body length of animal, 1.30 (Gaur <i>et al.</i> 2002)	$\text{m}$
$\tau_r$	Daily atmospheric transmittance	Dimensionless
$\omega$	Hour angle	$\text{rad}$
$\phi$	Latitude, 0.673	$\text{rad}$

Table 2. *Body-core component\**

Equation	Description
1.0. $d(M_b \times c_{pb} \times T_b)/dt = HE - A(q''_{cond, b-s} + q''_{resp, b-a})$	Energy balance body-core†, W
1.1. $q''_{cond, b-s} = \rho c_p / r_s (T_b - T_s)$	Conduction heat flux from body-core to skin†, W/m <sup>2</sup>
1.2. $r_s = \max\{\min\{(T_b - T_{bref}) \times (-30.83) + 50, r_{s,max}\}, r_{s,min}\}$	Resistance to heat transfer from body-core to skin‡, s/m
1.3. $q''_{resp, b-a} = V_t \times RR \times \rho c_p (T_{vb} - T_{va}) / A$ $+ \rho c_p [p_{w,sat}(T_b) - p_w(T_a)] / (\gamma \times r_{vl})$	Heat loss through respiration†§, W/m <sup>2</sup>
1.4. $V_t = 2.09 \times 10^{-5} \times M_b^{0.75} + 5.19 \times 10^{-4}$	Tidal volume  , m <sup>3</sup> /breath
1.5. $RR = \max\{(a_{rr} \times T_b - b_{rr}), RR_{min}\} / 60$	Respiration rate¶, breaths/s
1.6. $T_{vb} = T_b(1 + 0.38 \times p_{w,sat}(T_b) / P)$ $T_{va} = T_a(1 + 0.38 \times p_{w,sat}(T_a) / P)$	Virtual temperature of the body†, K Virtual temperature of the air†, K
1.7. $p_{w,sat}(T) = 6.11 \times 10^2 \times e^{17.27(T-273.2)/(T-35.86)}$	Saturation vapour pressure at temperature T**, Pa
1.8. $P = P_o \times e^{(-elev/H)}$	Atmospheric pressure††, Pa
1.9. $r_{vl} = [2.7 \times 10^{-6} \times RR + 5.0 \times 10^{-5}]^{-1}$	Resistance to water vapour transfer in lungst, s/m
1.10. $HE = H_e E + H_d E + H_r E$	Heat production, W
1.11. $H_e E = k_{maint} \times EBW^{0.75} \times Q_{10}^{(T_b - T_{bref})/10}$	Endogenous heat production‡‡, W
1.12. $H_d E = DMI_m (ME - NE_m) / 8.64 \times 10^4$	Heat energy of digestion, W
1.13. $H_r E = DMI_g (ME - NE_g) / 8.64 \times 10^4$	Heat energy from retained energy, W
1.14. $DMI_m = \frac{H_e E}{NE_m}$	Dry matter intake for maintenance, kg DM/s
1.15. $DMI_g = DMI - DMI_m$	Dry matter intake for gain, kg DM/s
1.16. $NE_g = 1.42 \times ME - 4.16 \times 10^{-8} \times ME^2$ $+ 6.96 \times 10^{-16} \times ME^3 - 6.91 \times 10^6$	Net energy for gain, J/kg DM
1.17. $NE_m = 1.37 \times ME - 3.30 \times 10^{-8} \times ME^2$ $+ 5.99 \times 10^{-16} \times ME^3 - 4.69 \times 10^6$	Net energy for maintenance, J/kg DM

\* All symbols, variables and parameters are described in Table 1.

† McArthur (1987).

‡ Finch (1985).

§ McGovern & Bruce (2000).

|| Empirically fit equation.

¶ Thompson *et al.* (2011).

\*\* Murray (1967).

†† Olson (1962).

‡‡ Eqns (1.10) to (1.17): (NRC 2000).

uniform, similar to Crosbie *et al.* (1961) and Stolwijk & Hardy (1966). The equations that represent the change in heat content of the body core, skin and coat layers (Eqns 1.0, 2.0 and 3.0, respectively; all in W), with respect to time are as follows:

$$d(M_b \times c_{pb} \times T_b)/dt = HE - A(q''_{cond, b-s} + q''_{resp, b-a}) \quad (1.0)$$

$$d(M_s \times c_{ps} \times T_s)/dt = A(q''_{cond, b-s} - q''_{cond, s-c} - q''_{evap, s-a}) \quad (2.0)$$

$$d(M_c \times c_{pc} \times T_c)/dt = 0 = A(q''_{cond, s-c} + q''_{solar, a-c} - q''_{conv, c-a} - q''_{lw rad, c-a}) \quad (3.0)$$

More detailed mathematical descriptions of the above equations are presented below and in Tables 2 to 4.

#### Body-core layer

The energy balance for the body core is described in Table 2. The heat fluxes are modelled using constitutive equations taken from the literature. The body-core heat content is a function of heat production (HE, W; W=J/s; Eqn 1.10, Table 2) and exchanges with the skin and environment by means of conduction and respiration, respectively.

The flow of heat from the body core to the skin,  $q''_{cond, b-s}$  (W/m<sup>2</sup>; Eqn 1.1) was modelled with

Table 3. *Skin layer component\**

Equation	Description
2.0. $d(M_s \times c_{ps} \times T_s)/dt = A (q''_{cond, b-s} - q''_{cond, s-c} - q''_{evap, s-a})$	Energy balance skin layer†, W
2.1. $q''_{cond, s-c} = \beta \times \rho c_p (T_s - T_c)/r_{ch}$	Heat flux via conduction from coat to skin‡, W/m <sup>2</sup>
2.2. $q''_{evap, s-a} = \min \left\{ \lambda \times a_{sr} \times e^{b_{sr} \times T_s} \right. \\ \left. \rho c_p [\rho_{w, sat}(T_s) - \rho_w(T_a)] / (\gamma \times r_v) \right\}$	Heat flux via evaporation through sweating from skin to ambient§   , W/m <sup>2</sup>
2.3. $r_v = (l - l_w)/(D (1 + 1.54((l - l_w)/d) T_{vdif}^{0.7}))$	Resistance to water vapor transfer¶, s/m
2.4. $l_w = \frac{cp \times u}{\left( cp \times \frac{u}{l} \right) + [r_{CH0}]^{-1}}$	Wind penetration of the coat <sup>g</sup> , m
2.5. $r_{CH0} = \left[ \frac{1}{r_d} + \frac{1}{r_b} + \frac{1}{r'_R} \right]^{-1}$	Resistance through the coat with no wind‡, s/m
2.6. $r_d = \frac{\rho c_p}{k_a} \times l$	Resistance of free convection‡, s/m
2.7. $r'_R = \frac{3 \times p \times l \times \rho c_p}{16 \times \sigma \times T_{mid}^3}$	Resistance of radiation**, s/m
2.8. $\beta = \frac{(1 - l/d) \tau + (d - 2 \times l)}{(\tau + d)}$	Body curvature parameter††, dimensionless
2.9. $r_{ch} = \left[ \frac{1}{r_{CH0}} + c_{pr} \times u \right]^{-1}$	Resistance through coat due to thermal properties††, s/m

\* All symbols, variables and parameters are described in Table 1.

† References: (McArthur 1987).

‡ Turnpenny *et al.* (2000a).

§ Thompson *et al.* (2011).

|| Monteith & Unsworth (2008).

¶ Cena & Monteith (1975).

\*\* Cena & Clark (1978).

†† McArthur & Monteith (1980).

a standard heat flow equation (McArthur 1987), which is a function of the temperatures of the body core and skin ( $T_b$  and  $T_s$ , K) and the resistance to heat transfer ( $r_s$ , s/m; Eqn 1.2). The effect of vasodilation/vasoconstriction is represented by  $r_s$ ; it has a linear relationship with the difference between the reference body-core temperature ( $T_{bref}$ , 311.65 K) and the actual body-core temperature and is constrained by minimum and maximum values (Finch 1985).

The heat loss through respiration,  $q''_{resp, b-a}$  (W/m<sup>2</sup>; Eqn 1.3), is split into convection and evaporation. The heat lost through respiratory convection is a function of tidal volume ( $V_t$ ; m<sup>3</sup>/breath; Eqn 1.4), respiration rate (RR; breaths/s; Eqn 1.5), the specific heat of air ( $\rho c_p$ ; 1220 J/m<sup>3</sup>/K), surface area ( $A$ ; m<sup>2</sup>) and the virtual temperatures of the body core and air ( $T_{vb}$  and  $T_{va}$ ; K) (McGovern & Bruce 2000). Virtual body core and air temperatures (Eqn 1.6) depend on saturation vapour pressure ( $\rho_{w, sat}(T)$ ; Pa; Eqn 1.7), atmospheric air pressure ( $P$ ; Pa; Eqn 1.8) and temperature ( $T$ ; K)

(McArthur 1987). The  $\rho_{w, sat}(T)$  function is evaluated through the Magnus Tetens equation (Murray 1967) for any temperature variable, i.e. the  $T$  argument may assume the values of  $T_b$  or  $T_s$ .

The calculation for  $V_t$  is derived from an equation by McGovern & Bruce (2000), which has a working temperature range from 15 to 40 °C. The genotype-specific parameter values used in the calculation of RR were obtained from a meta-analysis of a wide range of literature data (Thompson *et al.* 2011). Estimates of these parameter values are as follows: *B. taurus*,  $a_{rr}=0.6212$  and  $b_{rr}=192.7$ ; *B. indicus*,  $a_{rr}=0.7303$  and  $b_{rr}=227.2$  ( $a_{rr}$ , slope for respiration rate equation, breaths/K/min;  $b_{rr}$ , intercept for respiration rate equation, breaths/min).

The heat loss by evaporation from the lungs is modelled following McArthur (1987), using the psychrometer constant ( $\gamma$ ; 66 Pa/K), the saturation vapour pressure of the body core ( $\rho_{w, sat}(T_b)$ ; Pa; Eqn 1.7), the vapour pressure of the air ( $\rho_w(T_a)$ ;

Table 4. Coat layer component\*

Equation	Description
3.0. $d(M_c \times c_{pc} \times T_c)/dt = A (q''_{cond, s-c} + q''_{solar, a-c} - q''_{conv, c-a} - q''_{lw rad, c-a})$	Energy balance coat layer†, W
3.1. $q''_{conv, c-a} = \rho C_p / r_H (T_c - T_a)$	Heat flux via convection from coat to ambient†, W/m <sup>2</sup>
3.2. $r_H = \rho C_p \times d / (k_a \times Nu)$	Resistance to convection†, s/m
3.3. $Nu = \max(0.48 \times Gr^{0.25}, 0.24 \times Re^{0.6})$	Nusselt number‡, dimensionless
3.4. $Re = \rho \times u \times d / \mu$	Reynolds number‡, dimensionless
3.5. $Gr = \left[ \frac{9.81 \times P \times \text{abs}(T_s - T_a) + 61 \times}{\text{abs}(\rho_w(T_s) \times T_s - \rho_w(T_a) \times T_a)} \right] / (2.73 \times P \times \mu^2)$	Grashof number‡, dimensionless
3.6. $\mu = -3.39 \times e^{-11} \times T^2 + 6.77 \times e^{-8} \times T + 1.23 \times e^{-6}$	Dynamic viscosity of air§, N s/m <sup>2</sup>
3.7. $\rho = 1.42 \times 10^{-5} \times T^2 - 1.28 \times 10^{-2} \times T + 3.75$	Density of air§, kg/m <sup>3</sup>
3.8. $q''_{solar, a-c} = (1 - \rho_c) \times \left[ f_c \times R_{dir} + 0.5(R_{diff} + \rho_g(R_{dir} + R_{diff})) \right]$	Heat flux via solar radiation from ambient to coat†, W/m <sup>2</sup>
3.9. $q''_{lw rad, c-a} = \sigma \times \varepsilon_c \times \varepsilon_r (T_c^4 - T_r^4)$	Long wave radiation flux  , W/m <sup>2</sup>
3.10. $\varepsilon_r = (\varepsilon_{sky} + \varepsilon_g) / 2$	Emissivity of surroundings  , dimensionless
3.11. $\varepsilon_{csky} = 1.24 \left( \frac{\rho_w(T_a) \times 10^2}{T_a} \right)^{\frac{1}{7}}$	Clear sky emissivity¶, dimensionless
3.12. $\varepsilon_{sky} = clf + (1 - clf) \times \varepsilon_{csky}$	Sky emissivity**, dimensionless
3.13. $T_r = (T_{sky} + T_g) / 2$	Radiant temperature of environment  , K
3.14. $T_{sky} = T_a \left( 0.71 + 5.6 \times 10^{-3} \times T_{dp} + 7.3 \times 10^{-5} \times T_{dp}^2 + 0.013 \times \cos\left(15 \times St \times \frac{\pi}{180}\right) \right)^{0.25}$	Radiant sky temperature††, K
3.15. $T_g = (1.36 \times T_a - 2.36) \times (0.075 \times \ln(R_{gh}) - 0.56)$	Radiant ground temperature‡‡, K
3.16. $T_{dp} = \frac{237.7 (\ln(\rho_w(T_a)/0.61))}{17.27 - \ln(\rho_w(T_a)/0.61)} + 273.2$	Dew point temperature§§, K
3.17. $f_c = \frac{A_h}{A}$	Form factor‡, dimensionless
3.18. $A = 0.14 \times M_b^{0.57}$	Surface area   , m <sup>2</sup>
3.19. $A_h = \frac{\left(\frac{d}{2}\right)^2 \pi}{\cos(\pi/2 - \alpha)} + (\tau - d)d$	Shadow area of animal¶¶, m <sup>2</sup>
Solar radiation sub-model***	
3.20. $R_{dir} = R_{gh} - R_{diff}$	Direct radiation, W/m <sup>2</sup>
3.21. $R_{diff} = 0.3 (1 - \tau_r^m) R_{oh}$	Diffuse radiation, W/m <sup>2</sup>
3.22. $R_{gh} = \tau_r \times R_{oh}$	Radiation reaching the earth's surface, W/m <sup>2</sup>
3.23. $R_{oh} = J_o \times R \times \cos(\theta)$	Extraterrestrial radiation, W/m <sup>2</sup>
3.24. $\tau_r = R_{gd} / R_{od}$	Daily atmospheric transmittance, dimensionless
3.25. $R_{od} = 117.5 \times 10^6 \times R^2 (H_d \times \sin(\phi) \times \sin(\delta) + \cos(\phi) \times \cos(\delta) \times \sin(H_d)) / \pi$	daily extraterrestrial radiation, J/m <sup>2</sup> /d
3.26. $\delta = -23.5 \times \cos(2 \times \pi (D_j + 10) / 365) \pi / 180$	Solar declination, rad
3.27. $H_d = D_j / 24 \times \pi$	Half-day length, h
3.28. $D_j = \cos^{-1}(-\tan(\delta) \times \tan(\phi)) 180 / \pi \times 2 / 15$	Day length, h
3.29. $R = 1 + 0.033 \times \cos(2 \times \pi \times D_j / 365)$	Sun's radius vector, rad



Table 4. (Cont.)

Equation	Description
3.30. $\sin(\alpha) = \sin(\delta) \times \sin(\phi) + \cos(\delta) \times \cos(\phi) \times \cos(\omega)$	Altitude angle, rad
3.31. $\omega = 15(St - 12)\pi/180$	Hour angle, rad
3.32. $\theta = \pi/2 - \alpha$	Solar zenith, rad
3.33. $m = (P/P_o)/\sin(\alpha)$	Optical air mass number, dimensionless

\* All symbols, variables and parameters are described in Table 1.

† Turnpenny *et al.* (2000a).

‡ Monteith & Unsworth (2008).

§ Empirically fit equations.

|| da Silva (2000).

¶ (Brutsaert 1975).

\*\* Crawford & Duchon (1999).

†† Duffie & Beckman (1991).

‡‡ Turco *et al.* (2008).

§§ derived from Magnus Tetens equation; Murray (1967).

||| Brody (1945).

¶¶ Geometric calculation.

\*\*\* All solar radiation equations are from Duffie & Beckman (1991) and Oyarzun *et al.* (2007).

Pa; model input), and the resistance to water vapour transfer in the lungs ( $r_{vj}$ ; s/m; Eqn 1.9). McGovern & Bruce (2000) presented an equation for respiratory evaporative heat loss, but unlike that for convective loss, it greatly overpredicted heat loss. Therefore, McArthur's equation for respiratory evaporation is used in the model.

Heat production (HE; W; Eqn 1.10, Table 2) is modelled with the use of equations adapted from the NRC (1984), where  $H_eE$  is endogenous heat production (W),  $H_dE$  is heat energy of digestion (W), and  $H_rE$  is heat energy from retained energy (W; Eqns 1.10 to 1.13). Heat energy of digestion and from retained energy are calculated from metabolizable energy (ME; J/kg DM), net energies for gain ( $NE_{gr}$ , J/kg DM; Eqn 1.16) and maintenance ( $NE_m$ , J/kg DM; Eqn 1.17) of the diet (NRC 2000).

#### Skin layer

The equations for the skin layer sub-model are shown in Table 3. The flow of heat from the skin to the coat ( $q''_{cond, s-c}$ ; W/m<sup>2</sup>; Eqn 2.1) is a function of body curvature ( $\beta$ ; dimensionless; Eqn 2.8), resistance of flow due to coat thermal properties ( $r_{ch}$ ; s/m; Eqn 2.9) and wind speed ( $u$ ; m/s; Eqn W1.0 (Table 5); McArthur & Monteith 1980), where  $c_{pr}$  is penetrability of the coat by the wind ( $2.4 \times 10^{-5}$  m

and  $r_{CHO}$  is resistance of the coat in still air (s/m; Eqn 2.5).

#### Cutaneous evaporation

Latent heat loss from the surface of the animal is limited by either its sweating rate or its potential evaporation rate, depending on environmental conditions. Therefore, the minimum values of the functions for sweating and potential evaporation rates are used to model latent heat loss from cutaneous evaporation ( $q_{evap, s-a}$ ; W/m<sup>2</sup>; Eqn 2.2), where the upper equation represents the heat loss due to sweating rate and the lower represents the maximum heat loss given the potential evaporation rate, which is subject to environmental conditions.

The maximum evaporation rate is a function of temperature, wind and vapour pressure, where  $p_{w, sat}(T_s)$  (Pa; Eqn 1.7) is the saturation vapour pressure of the skin and  $r_v$  is the resistance to water vapour transfer (s/m; Eqn 2.3). The equation for  $r_v$  is a function of coat length ( $l$ ; m) and body-core diameter ( $d$ ; m) (Turnpenny *et al.* 2000b).

The sweating rate for the animal is modelled using an empirical equation that has species-dependent parameters obtained from a meta-analysis of a wide range of data from the literature (Thompson *et al.* 2011). This equation was obtained through regressions of sweating data v. skin temperature for both *B. taurus*

Table 5. Wind sub-model used in Tables 3 and 4\*

Equation	Description
W1.0. $u = u_m \left( \frac{h_x}{h_m} \right)^{0.14}$	Wind speed at mid-animal height†, m/s
W1.1. $h_x = 0.50 + 1.21 \times 10^{-3} \times M_b - 1.16 \times 10^{-6} \times M_b^2$	Mid animal height‡, m
W1.2. $u_m = \begin{cases} (WSX - WSN) \times \sin\left(\frac{\pi \times BBD}{D_l + 2 \times a}\right) + WSN, & (H_n + c \leq St < H_o) \\ WSN + (SSN - WSN) \times e^{\left(-b \times \frac{BBD}{N_l}\right)}, & (St < H_n + c \text{ or } H_o \leq St) \end{cases}$	Wind speed§, m/s
W1.3. $WSX = 1.58 + 1.18 \times DWT$	Maximum wind speed§, m/s
W1.4. $WSN = 0.19 + 0.58 \times DWT$	Minimum wind speed§, m/s
W1.5. $SSN = (WSX - WSN) \times \sin\left(\frac{\pi \times DDY}{D_l + 2 \times a}\right) + WSN$	Wind speed at sunset§, m/s
W1.6. $DDY = D_l - c$	Length of day wind speed§, h
W1.7. $BBD = \begin{cases} 24 - H_o + St, & (St < H_n + c) \\ St - (H_n + c), & (H_n + c \leq St < H_o) \\ St - H_o, & (H_o \leq St) \end{cases}$	Time from sunrise§, h

\* All symbols, variables and parameters are described in Table 1.

† Albright (1990).

‡ ASABE (2006).

§ Peterson & Parton (1983) and Hasson *et al.* (1990).

and *B. indicus*. The parameter values for *B. taurus* and *B. indicus* are  $a_{sr}=2.75 \times 10^{-18}$ ,  $b_{sr}=0.147$  and  $a_{sr}=3.93 \times 10^{-18}$ ,  $b_{sr}=0.222$ , respectively ( $a_{sr}$ , g/m<sup>2</sup>;  $b_{sr}$ , per K).

Skin thickness is used to calculate the skin mass. There are species differences in skin thickness (Pan 1963), but these had no impact on the heat balance for the two species, thus skin thickness was kept at a constant value.

#### Coat layer

The coat layer sub-model is shown in Table 4. Conduction loss through the ground is not included in this model. The animal is most likely to be standing while heat stressed and in this position there is minimal contact with the ground, making conduction loss negligible. Moreover, O'Connor & Spotila (1992) explain that conduction loss calculations are often difficult, rendering the results inaccurate, and thus are usually omitted from animal heat balance models.

The surface area of the animal,  $A$  (m<sup>2</sup>; Eqn 3.18), is calculated with the use of the equation developed for *B. taurus* (Brody 1945) and is a function of body-core mass ( $M_b$ ; kg) and an animal surface area

parameter,  $b_A$  (0.140 m<sup>2</sup>/kg<sup>0.57</sup>). Johnston *et al.* (1958) found that *B. indicus* have c. 12% more surface area than do *B. taurus*, so for *B. indicus* the parameter  $b_A=0.157$  m<sup>2</sup>/kg<sup>0.57</sup>.

#### Long-wave radiation

Long-wave radiation ( $q''_{lw\ radi}$ ; W/m<sup>2</sup>; Eqns 3.9 to 3.15) is a function of the emissivities and radiant temperatures of the animal and surroundings and the Stefan–Boltzmann constant ( $\sigma$ ; W/m<sup>2</sup>/K<sup>4</sup>). The emissivities in the infrared are given by  $\epsilon_{cr}$  for the surface of the animal (0.98; da Silva 2000), and by  $\epsilon_r$  for the environment, the latter being the average of the ground ( $\epsilon_g$ ; dimensionless; 0.98 for vegetation; da Silva 2000), and sky emissivities ( $\epsilon_{sky}$ ; dimensionless). Sky emissivities is calculated as a function of  $p_w(T_a)$ ,  $T_a$  (Brutsaert 1975) and cloud cover ( $clf$ ; index ranging from 0 to 1 where 0 represents clear skies and 1 represents complete cloud cover; Crawford & Duchon 1999).

The coat surface temperature and radiant temperature of the surroundings are given by  $T_c$  and  $T_r$  (K), respectively:  $T_r$  (Eqn 3.13) is calculated as the average of the sky ( $T_{sky}$ ; Eqn 3.14) and ground ( $T_g$ ; Eqn 3.15) radiant temperatures (K). Dew point temperature

( $T_{dp}$ ; K) is calculated with the use of  $p_w(T_a)$ , according to the Magnus Tetens equation (Murray 1967).

### Convection

Convection at the coat surface ( $q_{conv, c-a}$ ;  $W/m^2$ ; Eqns 3.1 to 3.7) is calculated with the temperature difference between the coat and air divided by the resistance to convective heat transfer ( $r_H$ ; s/m; Eqn 3.2). The  $r_H$ , which uses the Nusselt number ( $Nu$ ; dimensionless), is dependent on the animal size and decreases as wind speed increases (Turnpenny *et al.* 2000a);  $Nu$  (Eqn 3.3) is a function of both wind speed and animal geometry and is calculated with the use of Reynolds ( $Re$ , dimensionless; Eqn 3.4) and Grashof ( $Gr$ , dimensionless; Eqn 3.5) numbers (Monteith 1973; Incropera & DeWitt 1990; McGovern & Bruce 2000) For natural convection, where  $Gr > 16 \times Re^2$ ,  $Nu = 0.48 \times Gr^{0.25}$ . For forced convection, where  $Gr < 0.1 \times Re^{0.6}$ ,  $Nu = 0.24 \times Re^{0.6}$ . Otherwise, when wind speed is low,  $Nu$  will be the greater of the two above estimates.

The  $Re$  and  $Gr$  numbers are functions of wind speed  $u$  (m/s; Eqn W1.0, Table 5), animal diameter  $d$  (m), the dynamic viscosity of air  $\mu$  (N s/m<sup>2</sup>; Eqn 3.6), and the density of air  $\rho$  (kg/m<sup>3</sup>; Eqn 3.7). The viscosity and density of air were fitted to equations, both of which are a function of the mean of the skin and air temperatures (K).

The model operates with the assumption that values of daily measurements of minimum and maximum air temperature, maximum solar radiation, vapour pressure (at any time throughout the day) and daily wind travel (DWT) are known.

### Solar radiation

Solar radiation absorbed by the animal ( $q''_{solar, a-c}$ ;  $W/m^2$ ; Eqn 3.8) comes from three sources: direct radiation ( $R_{dir}$ ; Eqn 3.20), diffuse radiation ( $R_{diff}$ ; Eqn 3.21) and radiation reflected from the ground surface, which is a function of  $R_{dir}$  and  $R_{diff}$ , all in  $W/m^2$ . Absorbed radiation is a function of reflection coefficients for the animal and the ground,  $\rho_c$  and  $\rho_g$  (J reflected/J intercepted), respectively, and form factor ( $f_c$ ; dimensionless; Eqn 3.17). Form factor is a function of the animal's body angle in relation to the sun and its body dimensions (Monteith & Unsworth 2008) and is calculated as the quotient of the shadow area of the animal ( $A_h$ ; m<sup>2</sup>; Eqn 3.19) and  $A$ . The shadow area of the animal is a function of the animal's body length

( $\tau$ ; m),  $d$  and the altitude angle of the sun ( $\alpha$ ; rad; Eqn 3.30).

The solar radiation sub-model is shown in Table 3 (Eqns 3.20 to 3.33). The calculation of  $R_{dir}$  and  $R_{diff}$  first requires the calculation of daily extraterrestrial radiation,  $R_{od}$  (MJ/m<sup>2</sup>/d), with the use of solar declination ( $\delta$ ; rad; Eqn 3.26), latitude ( $\phi$ ; rad), half-day length ( $H_d$ ; rad; Eqn 3.27) and the correction factor for the variation of the earth–sun distance throughout the year ( $R$ ; rad; Eqn 3.29) (Duffie & Beckman 1991; Oyarzun *et al.* 2007).

Solar declination is the angle of the earth in relation to the sun (0° is reached twice a year at the equinox), based on the Julian day,  $D_j$  (1 Jan is 1 and 31 Dec is 365; d; Eqn 3.28). Altitude angle of the sun is the altitude of the sun at any given point throughout the day: at noon on the equator during the equinox it is 90° or  $\pi/2$ . Solar zenith ( $\theta$ ; rad) is the complement to  $\alpha$ . Hour angle ( $\omega$ ; rad; Eqn 3.31) is the daily rotation of the earth at any given hour (where midnight is 0), and is a function of standard time ( $St$ ; h), which is the hour of the day using a 24-h clock.

The extra-terrestrial radiation,  $R_{oh}$  ( $W/m^2$ ; Eqn 3.23) is calculated with the use of the solar constant,  $J_o$  (1350  $W/m^2$ ; Johnson 1954). The radiation reaching the earth's surface,  $R_{gh}$  ( $W/m^2$ ; Eqn 3.22) is then calculated with the use of the daily atmospheric transmittance ( $\tau_r$ ; dimensionless; Eqn 3.24) and  $R_{oh}$ .  $\tau_r$  is the ratio of the daily global radiation ( $R_{gd}$ ;  $J/m^2/d$ ), which is obtained from data collected through weather stations, to  $R_{od}$ . Cloud cover is accounted for through  $\tau_r$ , which decreases as cloud cover increases. Diffuse radiation increases in proportion to total radiation as cloud cover increases and  $\tau_r$  decreases.

Diffuse radiation (Eqn 3.21) is a function of  $\tau_r$ ,  $R_{oh}$ , and optical air mass number,  $m$  (dimensionless; Eqn 3.33). Atmospheric pressure and atmospheric pressure at sea level are given by  $P$  (Pa; Eqn 1.8) and  $P_o$  ( $1.01 \times 10^5$  Pa), respectively. Ground elevation is elev (500 m), and  $H$  is the height of the atmosphere (8000 m). Direct radiation (Eqn 3.20) is calculated as the difference between  $R_{gh}$  and  $R_{diff}$ .

### Air temperature

The equations used to calculate air temperature (Table 6) come from Cesaraccio *et al.* (2001) and from the variables calculated in the Solar Radiation section (Table 4). Figure 2 shows a sketch of the inputs needed for the air temperature sub-model, where all temperatures are in K and all of the times,  $St$ , are

Table 6. Air temperature sub-model used in Tables 2 to 4\* †

Equation	Description
$A1.0. T_a(St) = \begin{cases} T_0 + \frac{T_n - T_0}{\sqrt{H_n - H_0}} \sqrt{St - H_0}, & (0 < St \leq H_n) \\ T_n + (T_x - T_n) \times \sin\left[\frac{\pi}{2} \left(\frac{St - H_n}{H_x - H_n}\right)\right], & (H_n < St \leq H_x) \\ T_n + (T_x - T_0) \times \sin\left[\frac{\pi}{2} + \frac{\pi}{2} \left(\frac{St - H_x}{4}\right)\right], & (H_x < St \leq H_0) \\ T_0 + \frac{T_p - T_0}{\sqrt{H_p - H_0}} \sqrt{St - H_0} & (H_0 < St \leq 24) \end{cases}$	Air temperature at time $St$ , K
A1.1. $H_x = H_0 - 4$	The time of maximum temperature, h
A1.2. $H_n = 12 - D_j/2 - \text{timediff}$	Hour of sunrise, h
A1.3. $H_0 = 12 + D_j/2 - \text{timediff}$	Hour of sunset, h
A1.4. $\text{timediff} = (4(\text{stdmer} - \text{longd}) + \text{EoT})/60$	Difference between solar time and standard time, h
$A1.5. \text{EoT} = 229.2(7.5 \times 10^{-5} + 1.87 \times 10^{-3} \times \cos(B) - 0.032 \times \sin(B) - 0.015 \times \cos(2 \times B) - 0.041 \times \sin(2 \times B))$	Equation of time, min
A1.6. $B = \pi(D_j - 1)2/365$	Variable accounting for Julian day, rad
A1.7. $T_0 = T_x - c_t(T_x - T_p)$	Temperature at sunset for current day, K
A1.8. $T_0' = T_x' - c_t(T_x' - T_p')$	Temperature at sunset for previous day, K

\* All symbols, variables and parameters are described in Table 1.

† Eqns (A1.2)–(A1.6) are from Duffie & Beckman (1991), whereas the rest are from Cesaraccio *et al.* (2001).

in hours (h). The air temperature calculation,  $T_a(St)$  (Eqn A1.0), is separated into four different parts of the day: from midnight to sunrise ( $H_n$ ), from  $H_n$  to time of maximum air temperature ( $H_x$ ), from  $H_x$  to sunset ( $H_0$ ) and then from  $H_0$  to midnight.

The difference between solar time and standard time is *timediff* (h; Eqn A1.4). *timediff* is calculated with the use of equation of time (EoT; min; Eqn A1.5), standard meridian for local time zone (*stdmer*;  $120^\circ$  for Davis, CA) and longitude (*longd*;  $120.5^\circ$  for Davis, CA) (Duffie & Beckman 1991).

### Wind speed

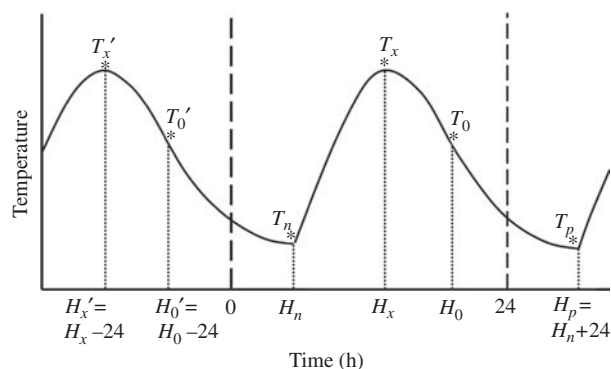
The wind speed sub-model, shown in Table 5, comes from Peterson & Parton (1983), with some revisions by Hasson *et al.* (1990). The inputs for wind speed calculations are DWT (m/s) and Julian day ( $D_j$ ). Maximum (WSX; m/s; Eqn W1.3) and minimum (WSN; m/s; Eqn W1.4) wind speeds are calculated from DWT (Hasson *et al.* 1990).

Wind speed at mid-animal height ( $u$ ; m/s; Eqn W1.0) is a function of wind speed ( $u_{mi}$ ; m/s) at height  $h_m$  (m)

and mid-animal height ( $h_x$ ; m). Wind speed is a two-part equation, which has day and night components (Eqn W1.2):  $u_m$  during the day (Eqn W1.2) is a function of WSX and WSN, the time from sunrise (BBD; h; Eqn W1.7) and two parameters  $a$  and  $c$ , which are the lag in maximum wind speed after noon ( $a=1.24$ ; h) and the lag in minimum wind speed after sunrise ( $c=1$ ; h), respectively. Wind speed at night is a function of WSN, the length of time that the wind speed is calculated during the day (DDY; h; Eqn W1.6), the coefficient that controls wind speed decrease at night ( $b=3.59$ ; dimensionless), day length ( $D_j$ ; h), night length ( $N_j=24 - D_j$ ; h) and the wind speed at sunset ( $SSN$ ; m/s; Eqn W1.5) (Hasson *et al.* 1990).

Wind speed at height  $h_m$ , is converted into  $u$  at  $h_x$  with the use of the equation by Albright (1990). Mid-animal height is calculated with the use of a regression equation, which was fit to *B. taurus* data and is multiplied by 1.2 for *B. indicus* (Eqn W1.1; ASABE 2006).

Both air temperature and wind speed sub-models had slope discontinuities, but this did not result in numerical difficulties in the solutions of the model and they compared well with hourly data.



**Fig. 2.** Sketch of the temperature model (Cesaraccio *et al.* 2001).  $T_n$  is the minimum temperature for current day,  $T_x$  is the maximum temperature for current day,  $T'_x$  is the maximum temperature for previous day,  $T'_0$  is the temperature at sunset for previous day and  $T_p$  is the minimum temperature for following day (all in K).  $H_n$  is the sunrise hour for current day,  $H_0$  is the sunset hour for current day,  $H'_0$  is the sunset hour for previous day,  $H_x$  is the time of maximum temperature for current day,  $H'_x$  is the time of maximum temperature for previous day and  $H_p$  is the sunrise hour for next day (all in standard time, h).

### Model simulation

The model was coded in Matlab (2010) and used a built-in ordinary differential equation solver (ode23t). It is capable of handling systems of differential algebraic equations (DAEs) (Shampine *et al.* 1999), such as Eqns 1.0, 2.0 and 3.0. The climate input data (temperature, vapour pressure, solar radiation and wind speed) were obtained from regional weather stations for 21–24 June 2006 in Davis, CA (CIMIS 2009) as daily inputs (Thompson *et al.* in press). Hourly weather data, if available, can be implemented in the model, replacing the climate equations with interpolation routines.

Animal inputs (shown in Table 7) are species-specific. *Bos indicus* have some adaptations which make them more tolerant to a tropical climate than *B. taurus*, such as a large dewlap, higher sweating rates, lighter coat, change in distribution of body reserves (large hump) and thicker skin (Pan 1963; Hansen 2004). The model accounts for the dewlap with increased surface area. Preliminary analysis showed that the thicker skin of *B. indicus* had no impact on heat balance. The differences in basal metabolic rate have little impact on heat balance (this was fully evaluated with the sensitivity analysis as described in Thompson *et al.* in press). The higher sweating rate and shorter coat are also both accounted

for in the animal inputs section. McDowell *et al.* (1958) found that the hump had no impact on heat regulation, thus this was not included in the model. The animal inputs also allow for a wide range of animal maturity from growing to mature cattle. This excludes pre-weaned calves as well as lactating and gestating animals, as such animals would require additional model changes, which may include sweating rates, heat of lactation and metabolic heat production.

### RESULTS AND DISCUSSION

As shown in Fig. 3, the patterns for air temperature and wind speed were similar, a pattern also found by Cesaraccio *et al.* (2001), Peterson & Parton (1983) and Hasson *et al.* (1990). The nadir in the diurnal fluctuations occurred at sunrise (or shortly thereafter in the case of wind speed) and the zenith some time after noon and before sunset. The inputs for this figure consisted of climate data from 21 to 24 June 2006 in Davis, CA (CIMIS 2009).

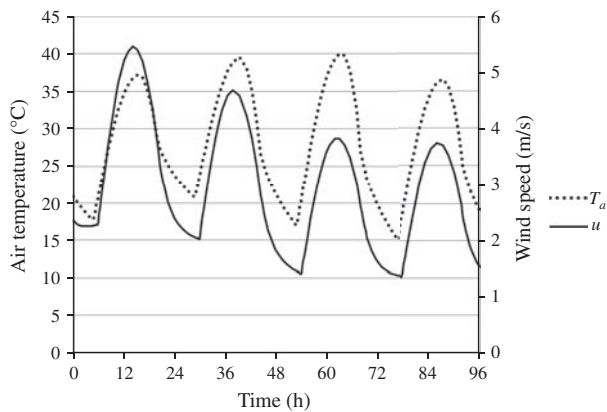
Figure 4 shows the ground level diffuse, direct and total radiation ( $\text{W}/\text{m}^2$ ). The peak for diffuse radiation was much flatter than those for direct and total radiation. In the early morning and late afternoon, the sun must travel through more of the earth's atmosphere, and the proportion of diffuse to direct radiation increases as more radiation is 'scattered' (Duffie & Beckman 1991). During the midday hours, although total radiation peaks, a lower proportion of the radiation is diffuse. On the fourth simulation day, the amount of total radiation and direct radiation fell, but the amount of diffuse radiation rose, due to the presence of cloud cover on that day. Cloud cover can increase the proportion of diffuse radiation from 0.1–0.3 to 1.0 of total radiation (Sen 2008).

The simulated evaporation potential (Eqn 2.2) behaved as expected, sharply decreasing at both high temperature and high humidity, as commonly seen in the humid tropics (Fig. 5). Evaporation potential increases as air temperature increases at low relative humidity, because absolute humidity potential increases with air temperature. However, at high relative humidity, the evaporation potential decreases as air temperature increases. At high relative humidity, the vapour pressure gradient decreases as air temperature nears body-core temperature, making it more difficult for the animal to lose water vapour to its surrounding environment.

The long-wave radiation and convection losses (Eqns 3.9 and 3.1, respectively) from the animal are

Table 7. Animal inputs for Nellore and Angus

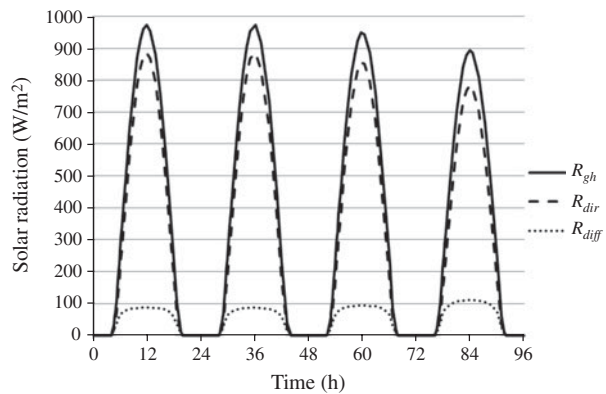
Parameter	Nellore	Angus	References
$d$ – body-core diameter (m)	$0.06 \times M_b^{0.39}$	$0.06 \times M_b^{0.39}$	(Ehrlemark 1988; Singh <i>et al.</i> 2002)
$M_b$ – body-core mass (kg)	400	400	–
$l$ – coat length (m)	0.002	0.004	(Hutchinson & Brown 1969)
$\rho_c$ – reflection coefficient of coat (dimensionless)	0.51	0.12	(Monteith 1973; da Silva 2008)
$a_{rr}$ – respiration rate parameter (breaths/K/min)	0.62	0.73	(Thompson <i>et al.</i> 2011)
$b_A$ – surface area parameter ( $\text{m}^2/\text{kg}^{0.57}$ )	0.14	0.16	(Brody 1945; Johnston <i>et al.</i> 1958)
$b_{rr}$ – respiration rate parameter (breaths/min)	192.7	227.2	(Thompson <i>et al.</i> 2011)
$a_{sr}$ – sweating rate parameter ( $\text{g}/\text{m}^2$ )	$2.75 \times 10^{-18}$	$3.93 \times 10^{-18}$	(Thompson <i>et al.</i> 2011)
$b_{sr}$ – sweating rate parameter (per K)	0.15	0.22	(Thompson <i>et al.</i> 2011)



**Fig. 3.** Air temperature ( $T_a$ ; °C) and wind speed ( $u$ ; m/s) over four days. Both lines follow similar diurnal fluctuations with nadirs at sunrise and zeniths in the early afternoon.

shown in Fig. 6 with increasing air temperature (wind speed is zero). The gradient between air and body-core temperatures drives these equations, which are independent of humidity. The difference between the temperature of the animal and its environment increases as air temperature decreases, which increases both the long-wave radiation and the convection losses from the animal. Above 36 °C, both long-wave radiation and convection became negative, indicating a net sensible heat gain by the animal, as also reported by Gebremedhin (1987). Even though Zhang *et al.* (2007) found that humidity increases convection losses, it does so only for objects with temperatures ranging from 77 to 177 °C, which is above the range for the surface of cattle. The increasing humidity resulted in water collecting on surfaces, which then evaporated, resulting in turn in greater sensible heat losses.

Figure 7 shows the surface plot of convective heat loss *v.* air temperature and wind speed. At low air temperature, increasing wind speed increases the



**Fig. 4.** Solar radiation on a horizontal surface at the ground. Radiation is separated into three lines, total, direct and diffuse ( $R_{gh}$ ,  $R_{dir}$  and  $R_{diff}$ , respectively).

convection loss experienced by the animal. Conversely, at high air temperature, increasing wind speed increases convection gain experienced by the animal. The results obtained by McArthur (1991) and Gebremedhin (1987) exhibited similar relationships among convective heat loss, air temperature and wind speed. Wind decreases resistance to convective heat transfer; thus, when the air temperature is low and the animal is losing heat due to convection, increasing wind speed will increase the rate of this loss, and when air temperature is high and the animal is gaining heat due to convection, increasing wind speed will increase the rate of this gain.

The model simulation outputs for *B. taurus* and *B. indicus* are shown in Fig. 8. The points for skin temperature lie between those for body-core and air temperatures, a relationship consistent with findings by Gebremedhin *et al.* (2008), Allen (1962) and Thomas & Pearson (1986). Maia *et al.* (2008) found coat temperatures to be between 2 and 14 °C above air temperature, which is consistent with the results

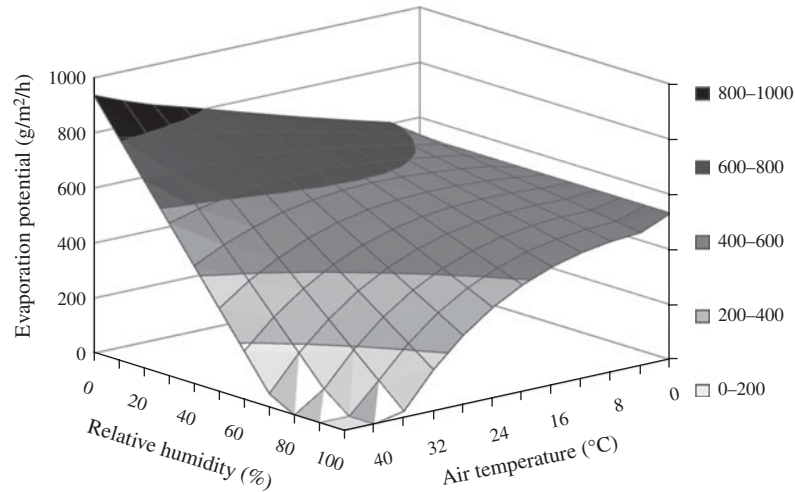


Fig. 5. Simulated evaporation potential of sweat on the skin of cattle v. air temperature and relative humidity.

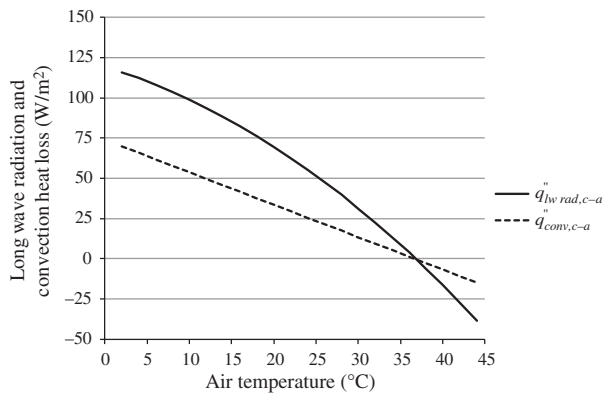


Fig. 6. Long-wave radiation ( $q''_{lw rad, c-a}$ ) and convection ( $q''_{conv, c-a}$ ) losses from cattle at zero wind speed over air temperature (°C).

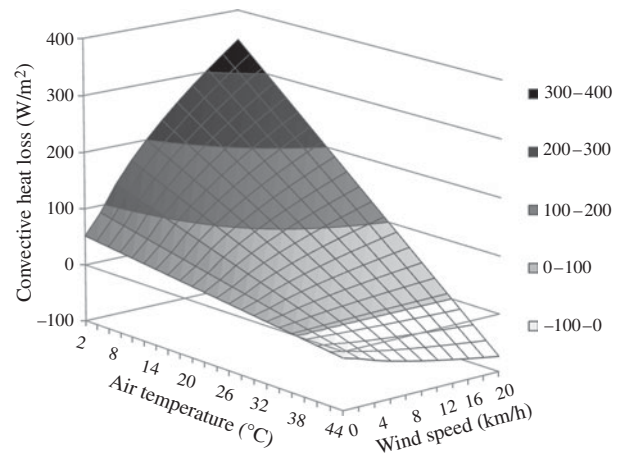
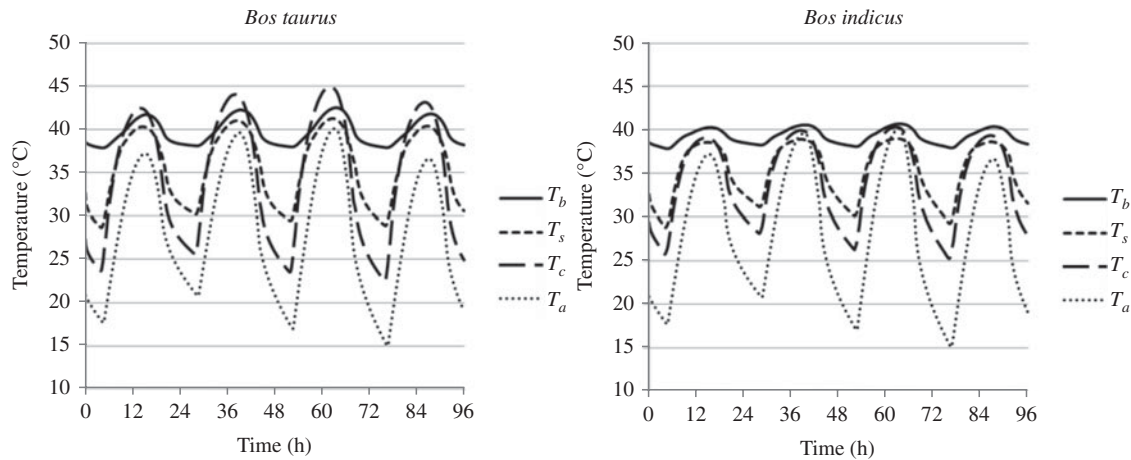


Fig. 7. Convective heat loss from the animal v. air temperature and wind speed.

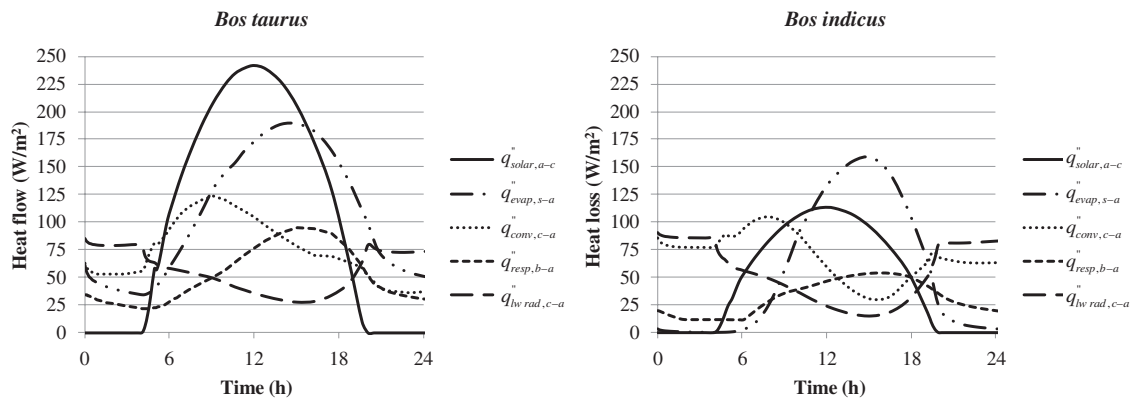
reported for *B. taurus* in the current work. The *B. indicus* body-core, skin and coat temperatures are all lower than the *B. taurus* due to their adaptations to hot climate conditions. For both species, the fluctuations in daily skin and coat temperatures are slightly offset from those in air temperature, due to the impact of solar radiation. Peak air temperature occurred around 15:00 h, while peak solar radiation occurred at midday. The increase in solar load on the animal resulted in an increase in coat temperature slightly before the increase in air temperature, and as soon as air temperature decreased, the solar load was no longer substantial, so the coat temperature could decrease along with air temperature.

Figure 9 shows the heat flows between the animal and its environment for *B. taurus* and *B. indicus*. For *B. taurus*, the highest incoming flow was solar

radiation absorption, which exhibited a peak during solar hours around noon. The inputs for this figure consisted of climate data from 21 to 22 June 2006, which includes the summer solstice, around which time incoming solar radiation is the highest for the year (reaching  $973 \text{ W/m}^2$  at 12:00 h). *Bos indicus* have lower solar radiation absorption due to higher coat reflectivity. For both species, cutaneous evaporation and respiratory losses account for the majority of heat loss during the day, especially near 16:00 h when air temperature reaches its maximum. Long-wave radiation and convection losses are greatest at night and in the early morning hours, as shown in studies by Kibler & Yeck (1959), Kelly et al. (1954) and Gebremedhin (1987), who found that, as air temperature decreases both convection and long wave radiation losses



**Fig. 8.** Simulation outputs for *Bos taurus* and *Bos indicus*. The model was simulated over four days (96 h). The outputs are body-core,  $T_b$ , skin,  $T_s$ , coat,  $T_c$  and air,  $T_a$ , temperatures.



**Fig. 9.** Heat flows between a *Bos taurus* and *Bos indicus* animal and its environment ( $W/m^2$ ). The flows are heat loss due to cutaneous evaporation,  $q_{evap,s-a}$ , solar absorption,  $q_{solar,a-c}$ , convection losses,  $q_{conv,c-a}$ , long wave radiation losses,  $q_{lw rad,c-a}$  and respiration losses,  $q_{resp,b-a}$ .

increase, whereas evaporative loss decreases. *Bos taurus* and *B. indicus* exhibit similar trends, although the flows for *B. taurus* are generally higher due to higher heat stress and thus increased need to dissipate excess heat.

### CONCLUSION

Future development for the current model could be the addition of a dynamic heat production component, which includes eating behaviour, a detailed representation of cloud cover and rainfall, a sprinkler/mister component, which includes droplet size and a heat production component of gestation and lactation.

Although many parts of this model can be evaluated independently, the prediction potential and extensive model analysis will be presented in the subsequent

paper (Thompson *et al.* in press). The current model integrates concepts and data relating to heat flows between animals and their environment. The equation forms and parameters are largely based on generally accepted physical, anatomical and physiological principles. Even though this model must be further evaluated against more complete, extensive datasets to assess its ability to predict body-core temperature in response to different environmental conditions for both *B. taurus* and *B. indicus*, the results produced by the model are physiologically sound and follow the trend of known biology.

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